NON-BLIND IMAGE RESTORATION WITH SYMMETRIC GENERALIZED PARETO PRIORS

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Introduction

- Non-Blind Image Restoration

\[ y = x \otimes k + n \]

- Assuming known \( k \) & noise level, estimate \( x \) from \( y \)
- An ill-posed problem, requiring priors on \( x \)

- Challenges
  - Find good image priors
  - Develop efficient numerical solutions

Our Proposal

- A New Parametric Image Prior
- A Fast & Effective Image Restoration Method
  - Closed-form numerical solutions
  - State-of-the-art image restoration quality & processing speed

The SGP Prior

- Symmetric Generalized Pareto (SGP)

\[ p(x | \omega, \gamma) = \frac{\omega \gamma^\omega x^{\omega-1}}{2(|x| + \gamma)^{\omega+1}}, x \in \mathbb{R} \]

- Symmetrizes generalized pareto to the whole real line
- Tail heaviness is controlled by \( \omega, \gamma > 0 \)

- A Fitting Example

- Tails are better captured by the SGP model

- Quantitative Evaluation

  - 100 images from van Hateren’s data set
  - 6 band-pass filter responses
  - Average likelihood scores
  - SGP comparable to hyper-Laplacian

Restoration with SGP

- The Maximum-a-posterior (MAP) Formulation

\[
\min_x \sum_i \left( \frac{\lambda}{2} (y - x \otimes k)^2 + \sum_j \log(|x \otimes f_j| + \gamma) \right)
\]

- Difficult to solve due to the non-differentiable regularizers

- Half-Quadratic Splitting Solution

  Decouples \( x \otimes f_1, x \otimes f_2 \) from SGP regularizers using auxiliary variables \( z_1, z_2 \) and quadratic penalty terms

\[
\min_{x, z_1, z_2} \sum_i \left( \frac{\lambda}{2} (y - x \otimes k)^2 + \sum_j \frac{\beta}{2} (x \otimes f_j - z_j)^2 + \sum_j \log(|z_j| + \gamma) \right)
\]

- Solves two sub-problems using block coordinate descent
  - Fixed \( z_1, z_2 \), solve \( x \) with 2D FFTs and IFFTs
  - Fixed \( x \), solve \( z_1, z_2 \) in a common 1D form independently on each pixel

\[
\min_z g(z) = \frac{\beta}{2} (z - v)^2 + \log(|z| + \gamma)
\]

Quadratic penalty terms, \( \beta \to \infty \)

Note that when \( z > 0 \):

\[
g'(z) = 0 \Rightarrow \beta (z - v)(z + \gamma) + 1 = 0
\]

Experimental Results

- Experimental Settings
  - 12 grayscale images, 10 blurring kernels, 3 noise levels
  - Comparison to L1 [Wang et al., SIAM JIS 2008], LUT [Krishnan and Fergus, NIPS 2009], GISA [Zuo et al., ICCV 2013]

- Quantitative Results
  - Average PSNR with 4 Gaussian kernels and 3 noise levels

\[
\begin{array}{cccc}
\text{Kernel} & \text{L1} & \text{SGP} & \text{LUT} & \text{GISA} \\
\hline
\text{1x1} & 24.91 & 25.42 & 26.48 & 30.51 \\
\text{2x2} & 23.91 & 24.82 & 25.75 & 29.87 \\
\text{3x3} & 22.91 & 23.79 & 24.64 & 28.75 \\
\text{4x4} & 21.91 & 22.64 & 23.43 & 27.75 \\
\text{5x5} & 20.91 & 21.54 & 22.29 & 26.75 \\
\end{array}
\]

- Average PSNR with 6 motion kernels and 3 noise levels

\[
\begin{array}{cccc}
\text{Kernel} & \text{L1} & \text{SGP} & \text{LUT} & \text{GISA} \\
\hline
\text{1x1} & 26.91 & 27.41 & 28.51 & 32.51 \\
\text{2x2} & 25.91 & 26.41 & 27.46 & 31.51 \\
\text{3x3} & 24.91 & 25.32 & 26.36 & 29.51 \\
\text{4x4} & 23.91 & 24.36 & 25.32 & 28.51 \\
\text{5x5} & 22.91 & 23.41 & 24.36 & 27.51 \\
\end{array}
\]

- Visual Comparison
  - The ‘Barbara’ example: 27x27 motion kernel + 10% noise

- Running Time (in Seconds)
  - The z-step: L1, SGP - 0.014 , LUT - 0.048, GISA - 0.023
  - The x-step: 0.576

Acknowledgement
This work is supported by the National Science Foundation under Grant Nos. IIS-0953373 and CCF-1319800, and National Institute of Justice Grant No 2013-IJ-CX-K010.